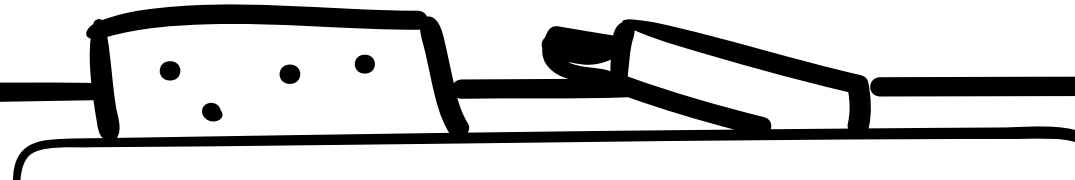
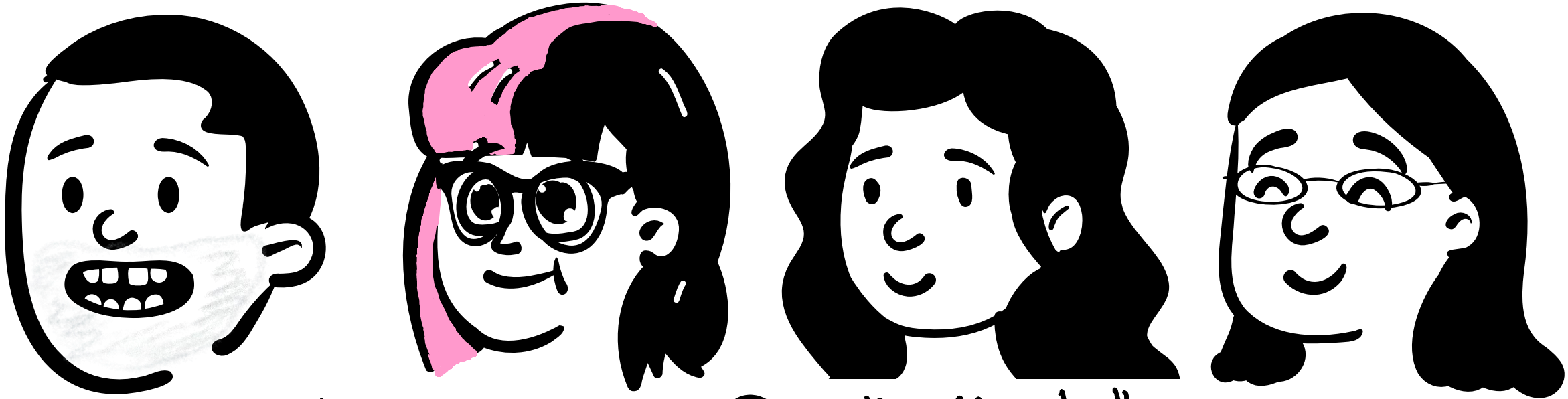
A Mixed Linear and Graded Logic

Victoria Vollmer
University of Kent



Joint work with some really cool people



Harley Eades III ME Danielle Marshall Dominic Orchard

1 ◦ Augusta University, 3 ◦ University of Kent ⊢

So what's a graded logic?

traditional vs. graded

$\Box A$

Modality formula

$\Box_r A$

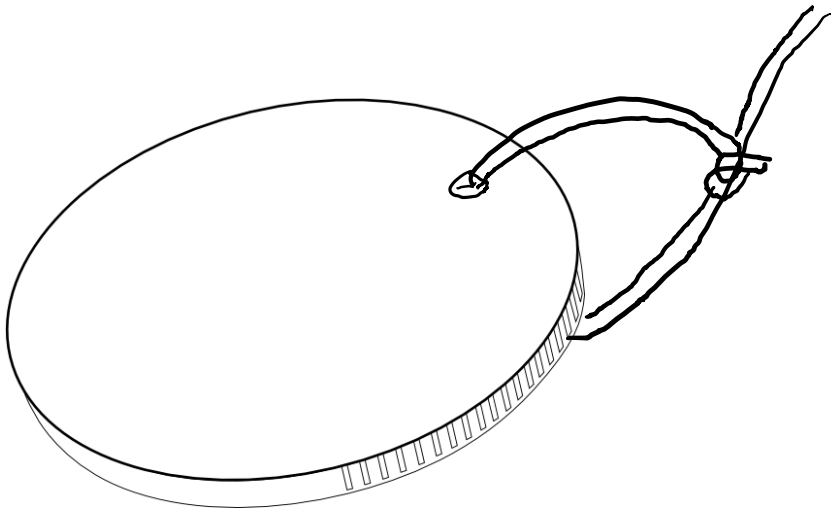
Modality is indexed

$r \in (R, \times, +, 1, 0, \leq)$

ordered algebra

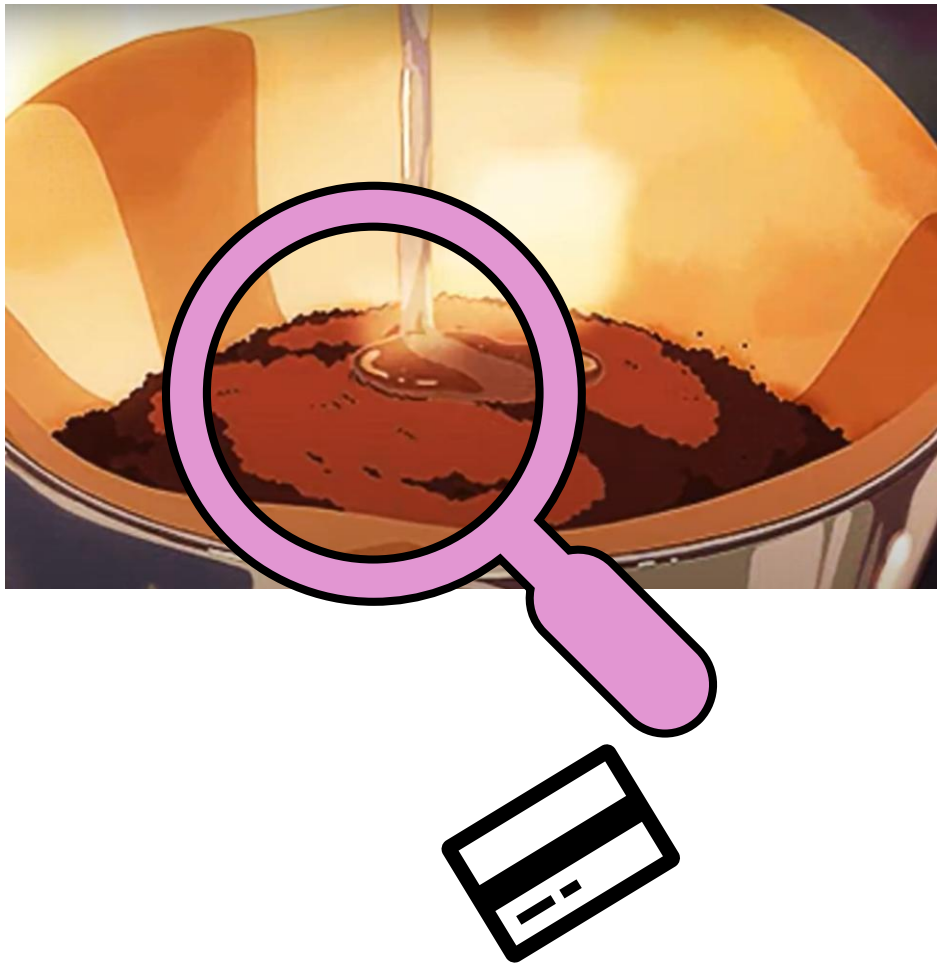
Okay, but why?

resources used exactly once

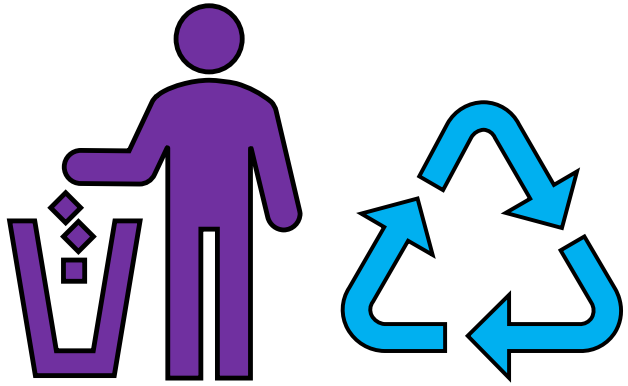


of-course

Okay, but why?

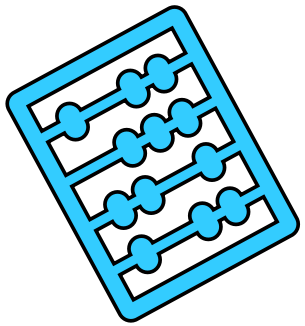


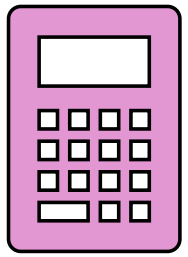
Examples!

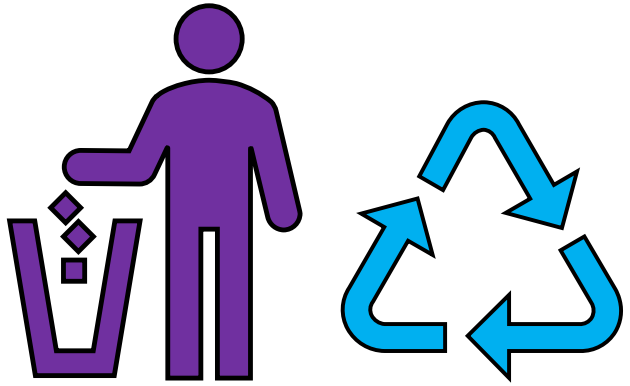


discard vs. Multiple Use

Examples!

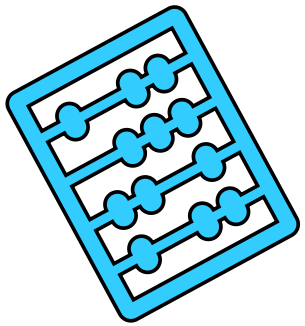


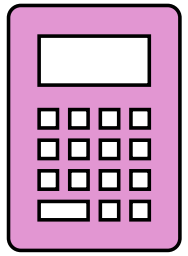


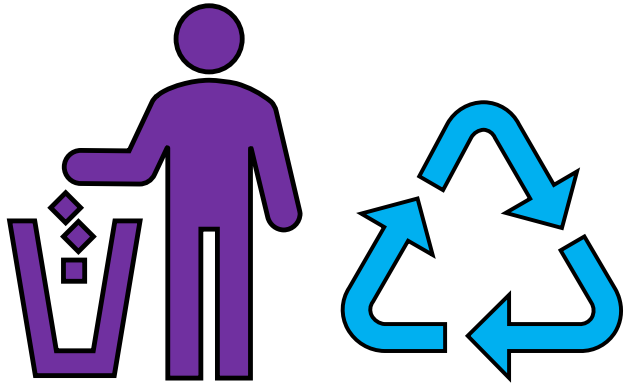


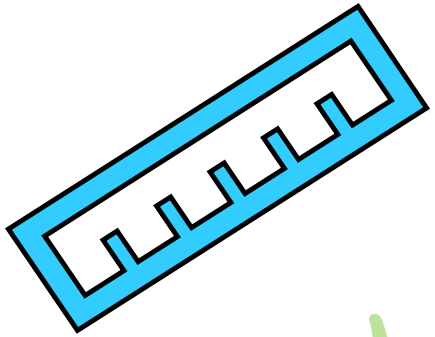
Usage Counting

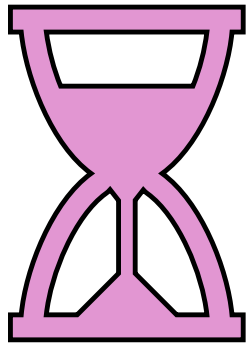
Examples!





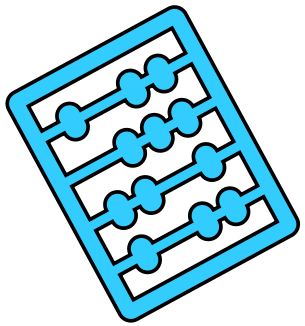


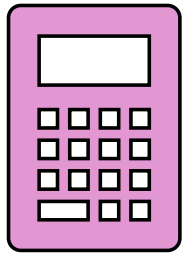


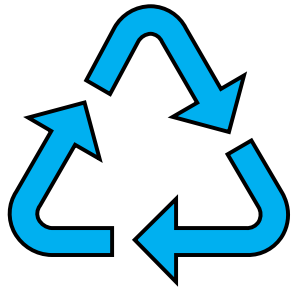


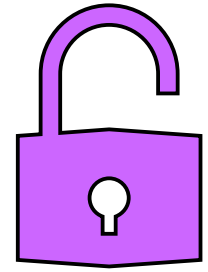
Space & time Complexity

Examples!

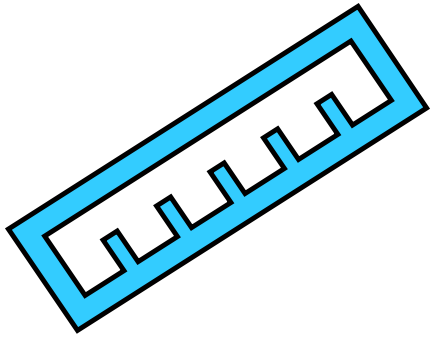


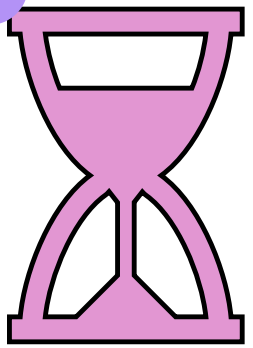




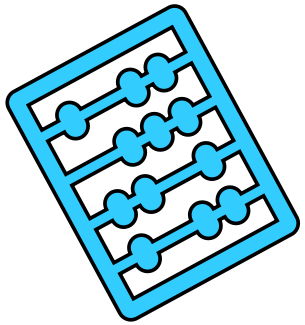


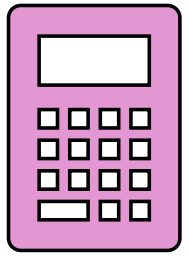
Security / Privacy

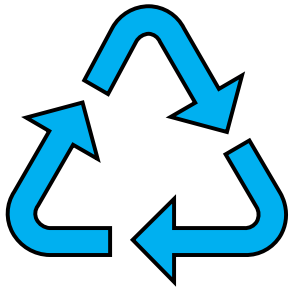


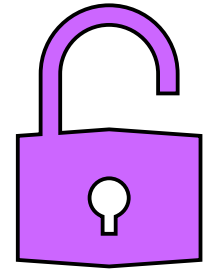


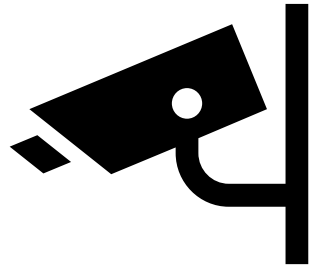
Examples!

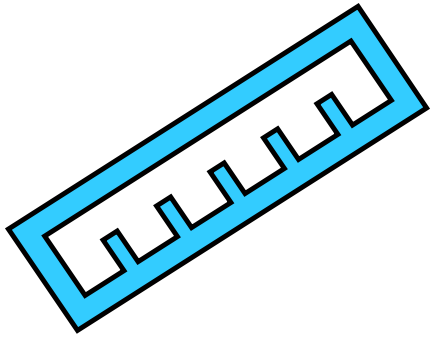


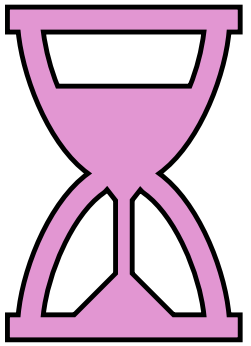


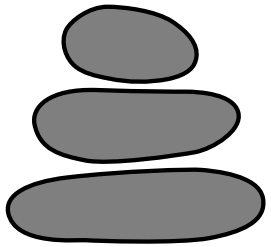


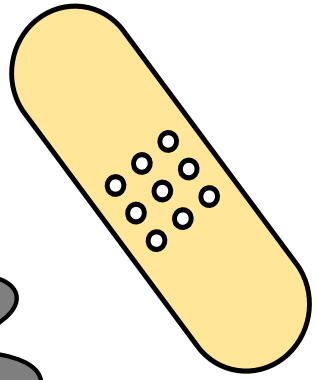












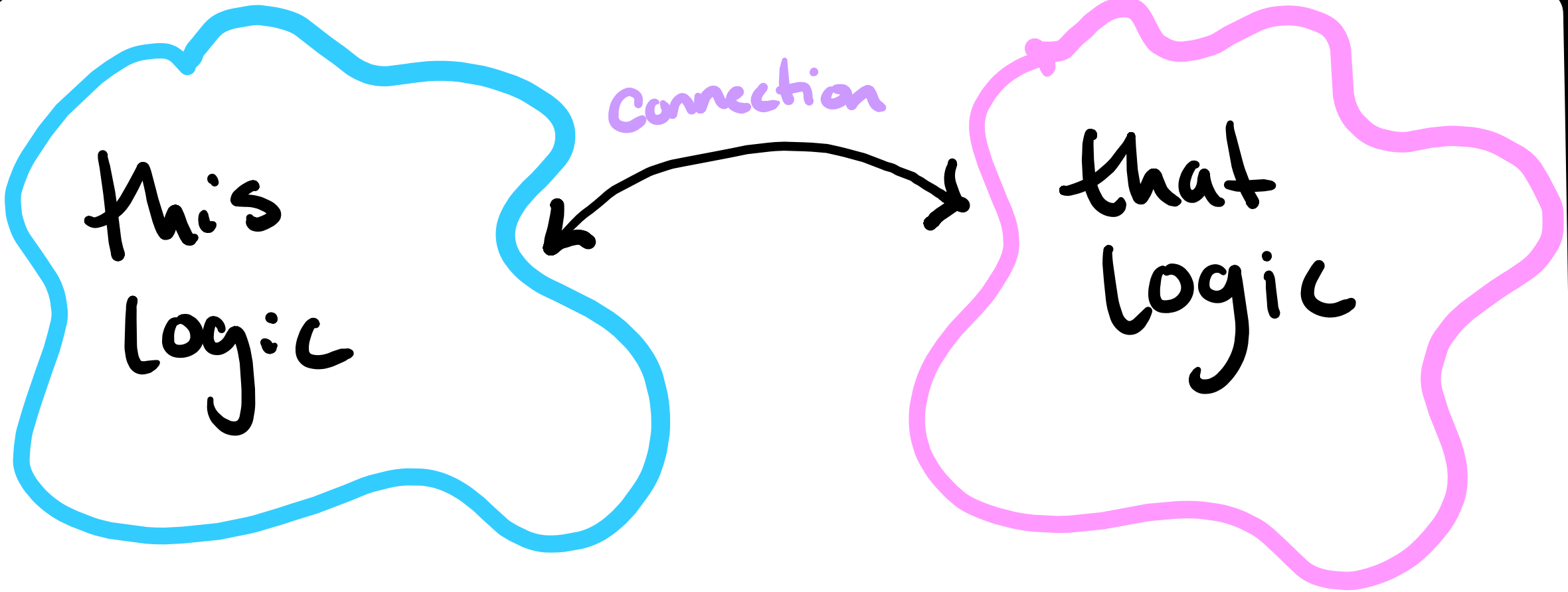
Error sensetivity

How do you even "Mix" a Logic?

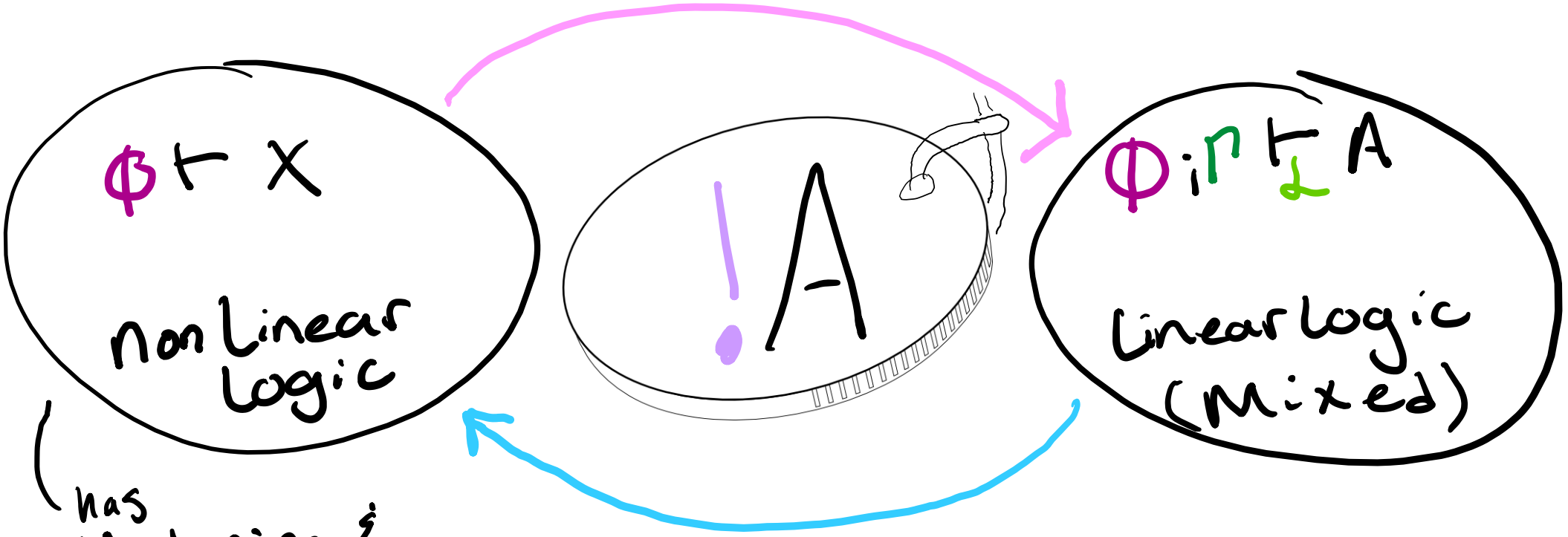
~~How~~ do you even "M:x" a log:c?

Why do you even "M:x" a log:c?

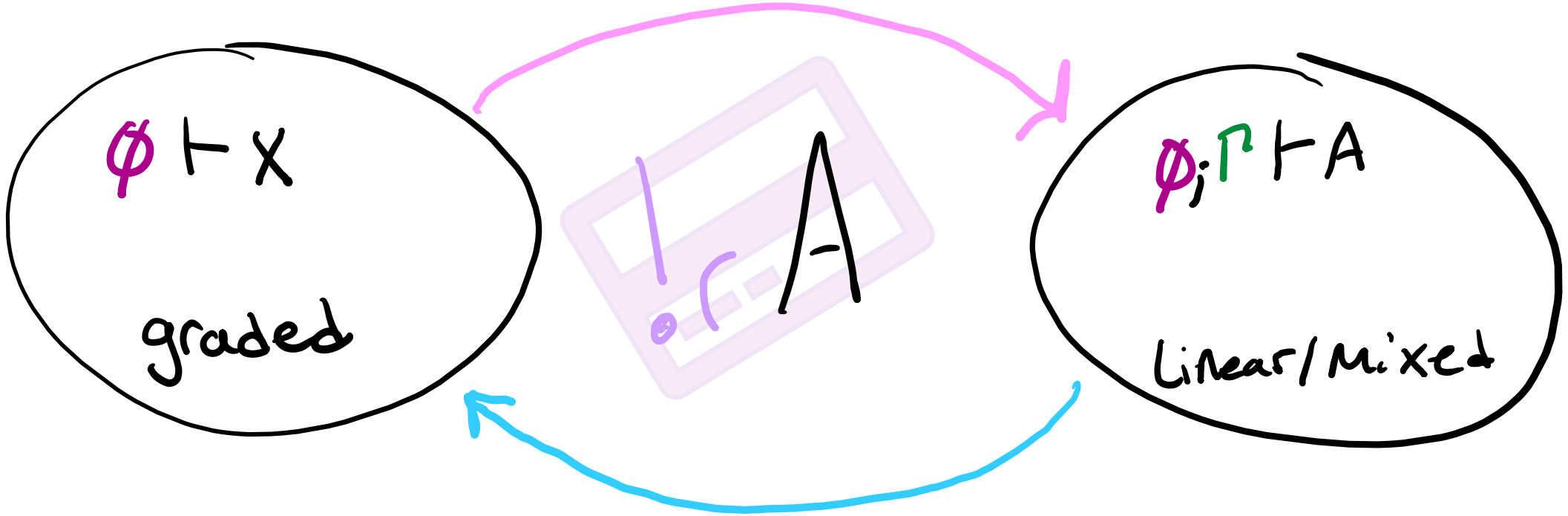
Why do you even "Mix" a Logic?



Benton's LNL


Φ ⊢ X
non Linear Logic
!A
Φ ; Γ ⊢ L A
Linear Logic (Mixed)
has Weakening & Contraction

What about with grades?



us future

Showing of our logic

graded

$X, Y, Z ::= J \mid X \boxtimes Y \mid \mathrm{Lin}(A)$

$\Delta ::= (X, \Delta)$ $\delta ::= (r, \delta)$

$r \circ X$ and $\delta \circ \Delta$

$\delta \circ \Delta \vdash_{GS} X$

Linear/Mixed

$A, B, C ::= I \mid A \otimes B \mid \mathrm{Grd}_r X$

$r \in (R, *, 1, +, 0, \leq)$

$\delta \circ \Delta ; \Gamma \vdash_{MS} A$

So why is the index an algebra?

Structural rules/properties

Contraction, Weakening, approximation, Cut

\+ identity ordering *

Showing of our logic

$$\frac{\S \odot \Delta ; \varphi \vdash_{MS} B}{\S \odot \Delta \vdash_{GS} \mathrm{lin}\, B}\ \mathrm{lin}_R$$

$$\frac{\S \odot \Delta ; A, \Gamma \vdash_{MS} B}{\S, 1 \odot \Delta, \mathrm{lin}(A) ; \Gamma \vdash_{MS} B}\ \mathrm{lin}_L$$

Showing of our logic

$$\mathrm{Grd}_r\ \frac{\Sigma \circ \Delta \vdash_{GS} X}{\Gamma * \Sigma \circ \Delta \mid \emptyset \vdash_{MS} \mathrm{Grd}\, r\, X}$$

$$\mathrm{Grd}_l\ \frac{\Sigma, r \circ \Delta, X \mid \Gamma \vdash_{MS} B}{\Sigma \circ \Delta \mid \mathrm{Grd}_r\, X, \Gamma \vdash_{MS} B}$$

all mixed up

Weakening

$$\frac{\Sigma_1, \Sigma_2 \circ \Delta_1, \Delta_2 \;|\; \Gamma \vdash_{MS} A}{\Sigma_1, \Sigma_0, \Sigma_2 \circ \Delta_1, X, \Delta_2 \;|\; \Gamma \vdash_{MS} A}$$

Contraction

$$\frac{\Sigma_1, \tau_1, \tau_2, \Sigma_2 \circ \Delta_1, X, X, \Delta_2 \;|\; \Gamma \vdash_{MS} A}{\Sigma_1, \tau_1 + \tau_2, \Sigma_2 \circ \Delta_1, X, \Delta_2 \;|\; \Gamma \vdash_{MS} A}$$

all mixed up

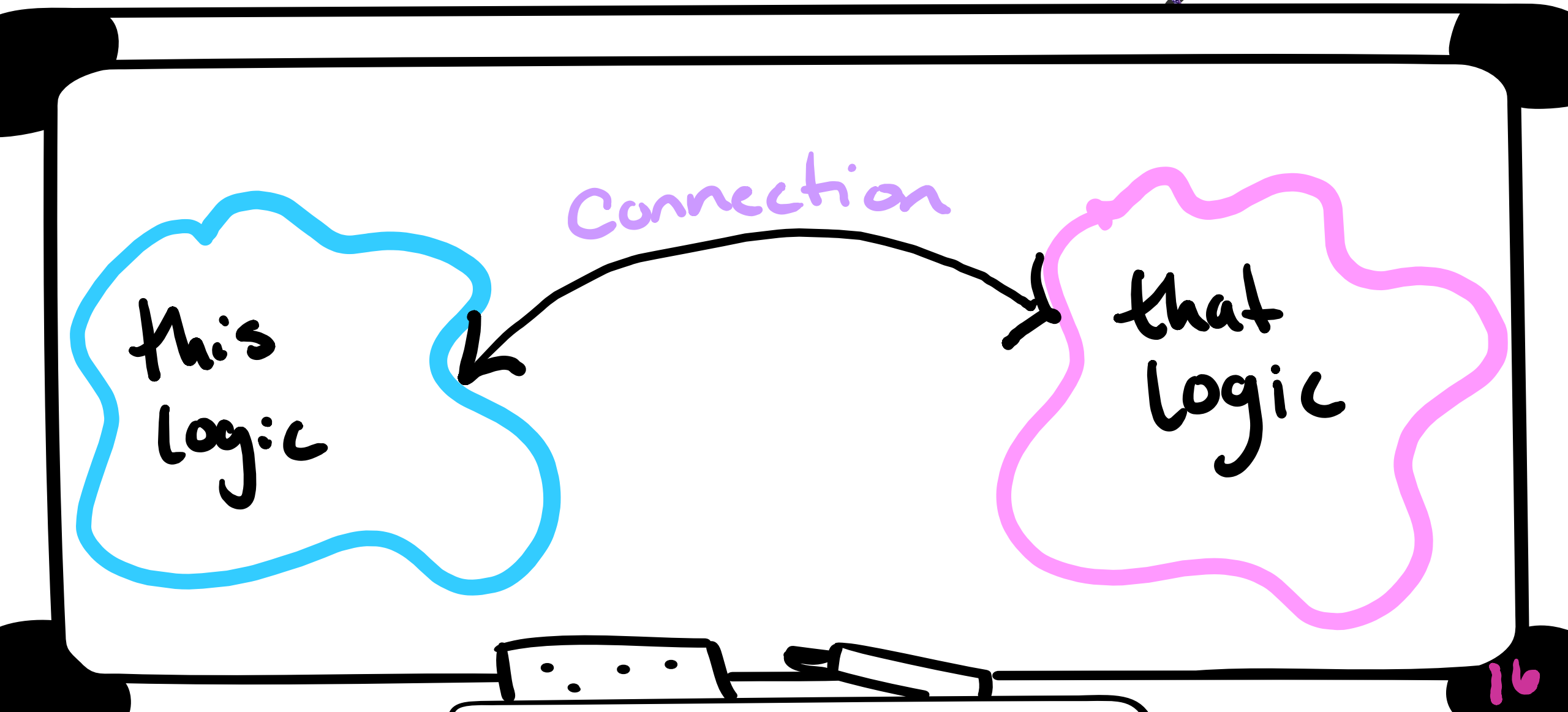
Graded Unit

$$\frac{\Sigma_1, \Sigma_2 \odot \Delta_1, \Delta_2 \; j \; \Gamma \vdash_{MS} A}{\Sigma_1, 1, \Sigma_2 \odot \Delta_1, J, \Delta_2 \; j \; \Gamma \vdash_{MS} A}$$

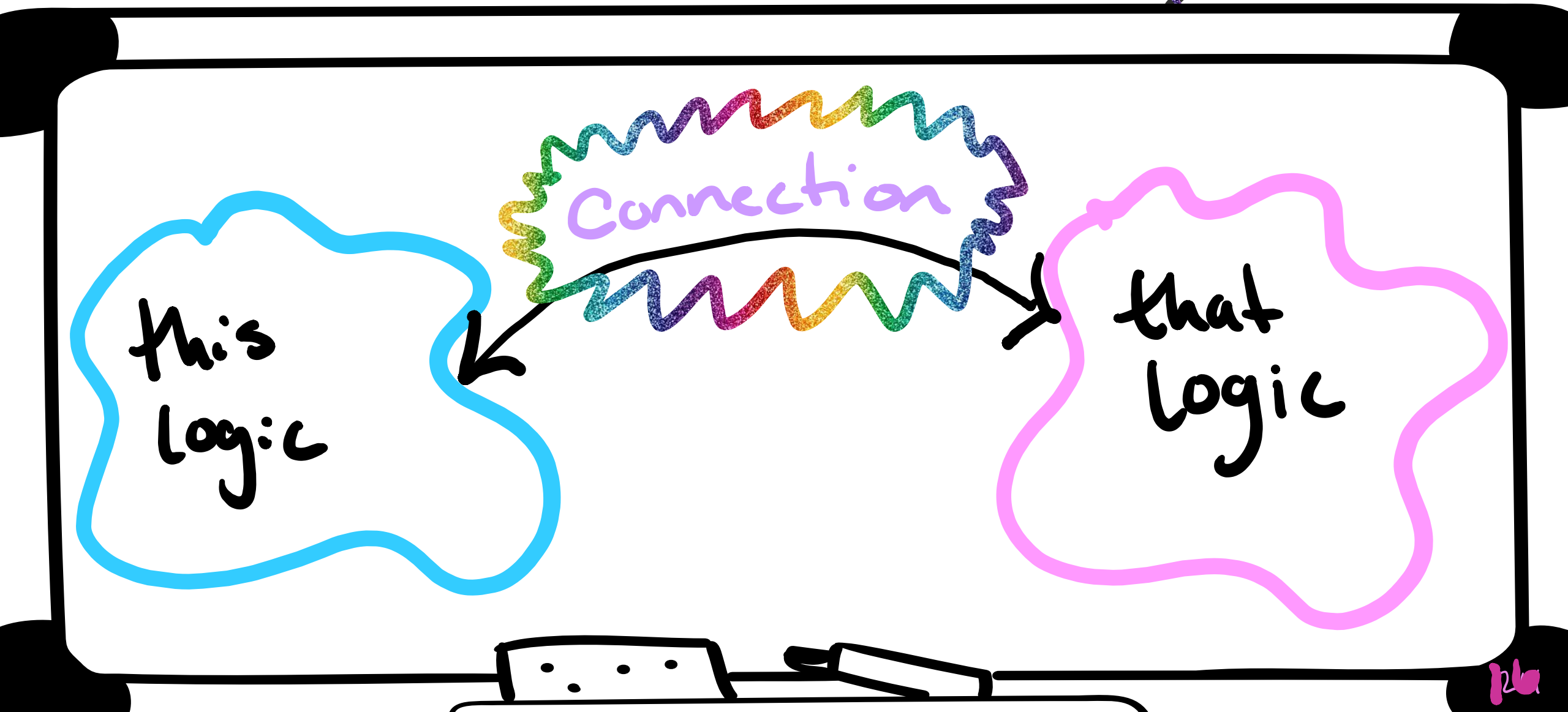
Graded tensorL

$$\frac{\Sigma_1, r, r, \Sigma_2 \odot \Delta_1, X, Y, \Delta_2 \; j \; \Gamma \vdash_{MS} A}{\Sigma_1, r, \Sigma_2 \odot \Delta_1, X \boxtimes Y, \Delta_2 \; j \; \Gamma \vdash_{MS} A}$$

But what does it all mean?



But what does it all mean?



How do you even "Mix" a Logic?

of course!

With a Linear exponential Comonad

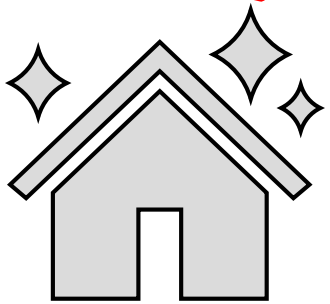
and some helpful facts about

Adjunctions (ok, actually I'll just talk about adjoint functors)

Benton 94

A-what-ion?

Town A

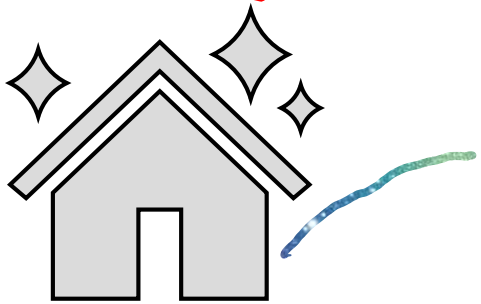


Station X

Town B

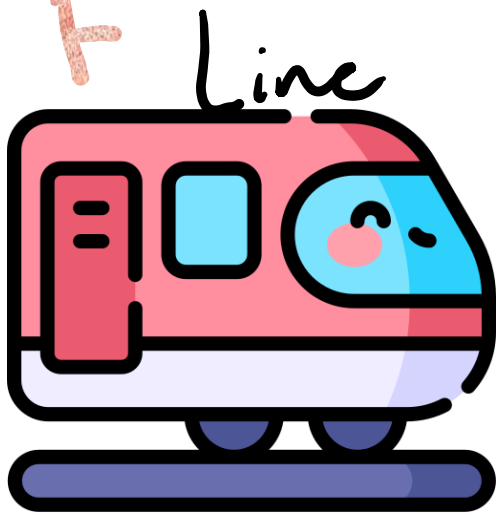
A-what-ion?

Town A

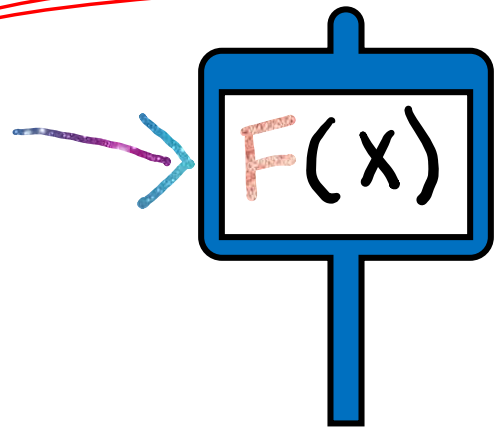


Station X

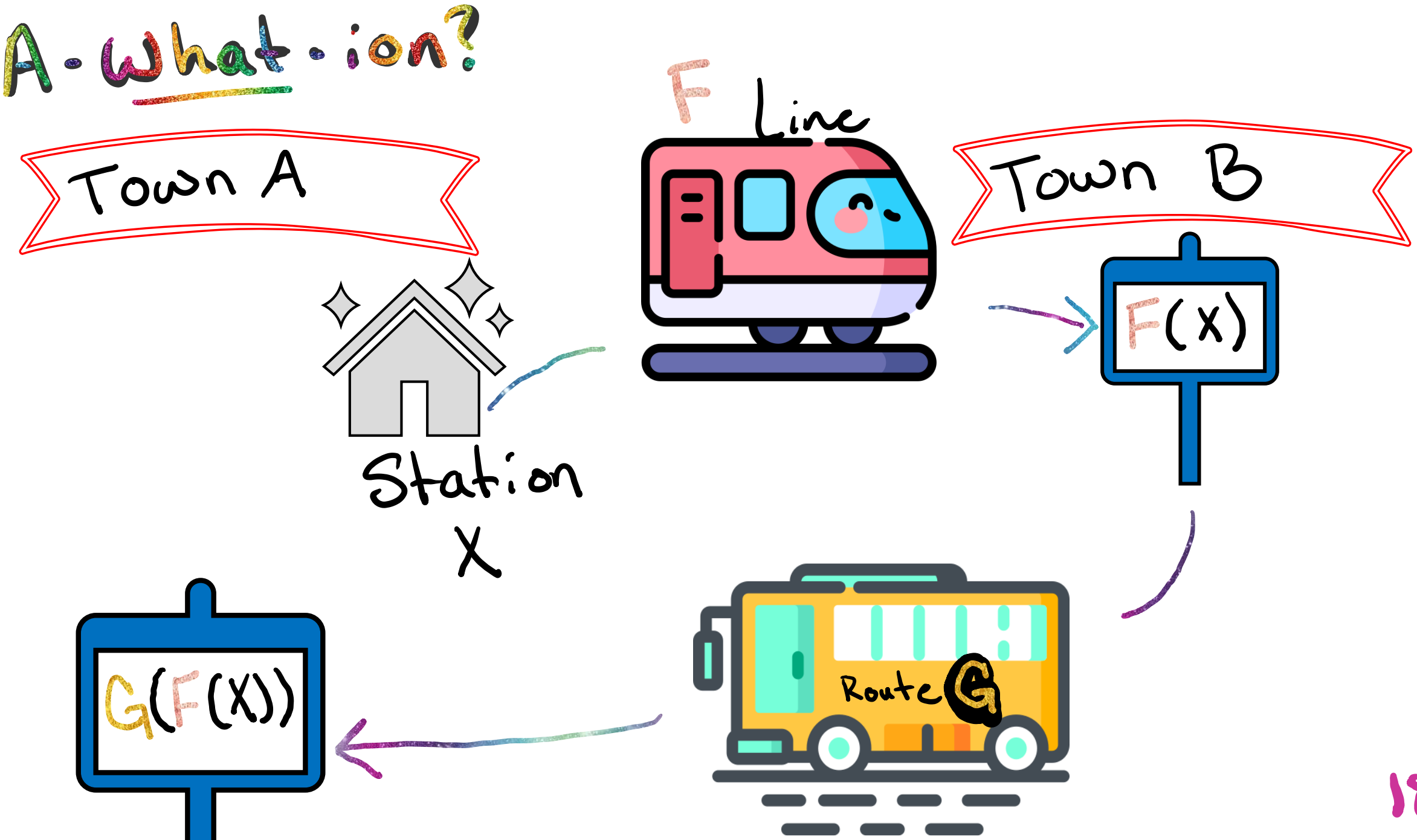
F Line



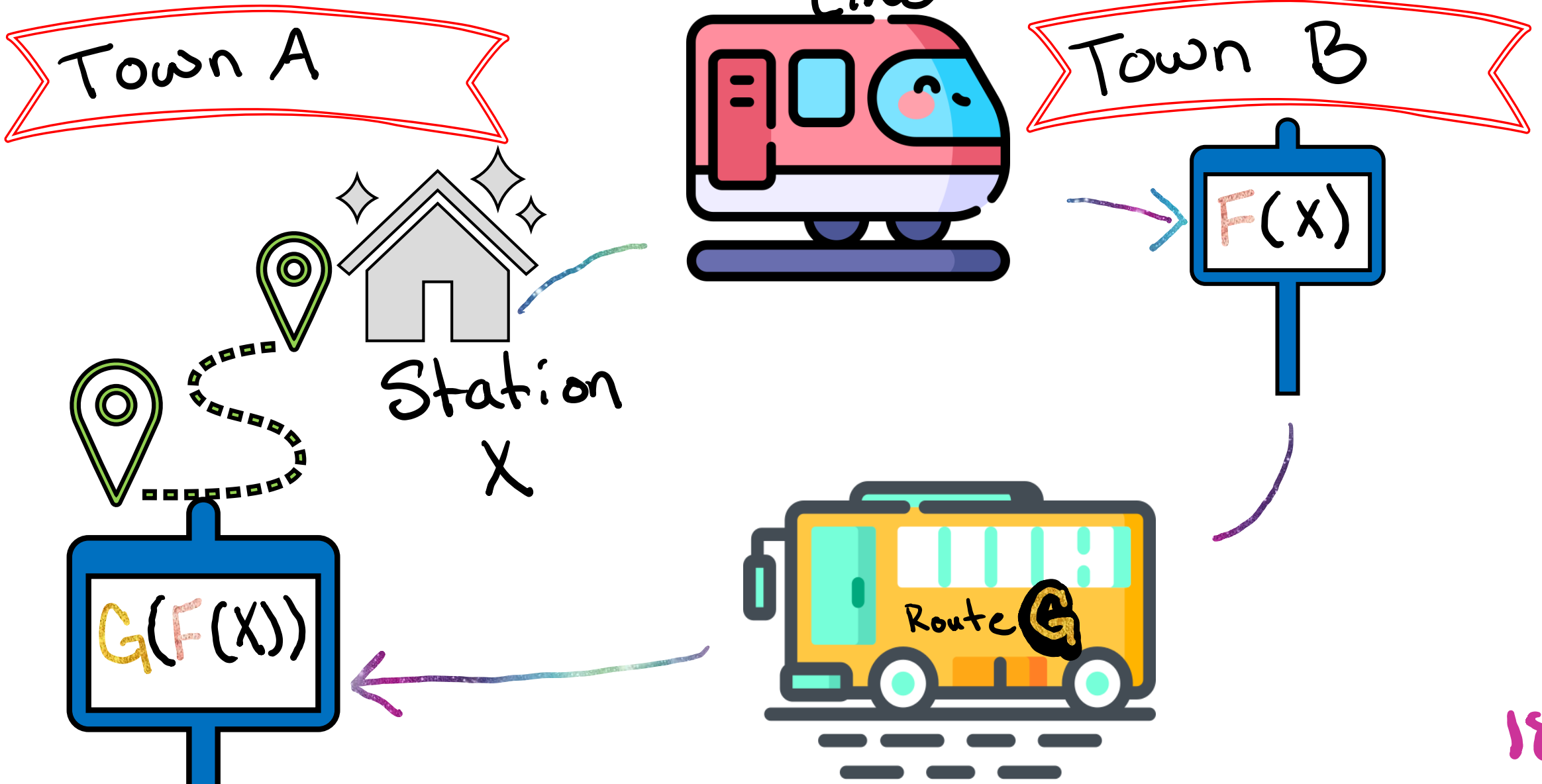
Town B



A-what-ion?



A-what-ion?



A·what·ion?

$F \dashv G$

$F : C \to D$

$G : D \to C$

Unit

$\eta : id_C \to G \circ F$

Counit

$\epsilon : F \circ G \to id_D$

Back up, Linear exponential huh?

One way to Model of-course from ILL is with a linear exponential comonad (comonad + properties)

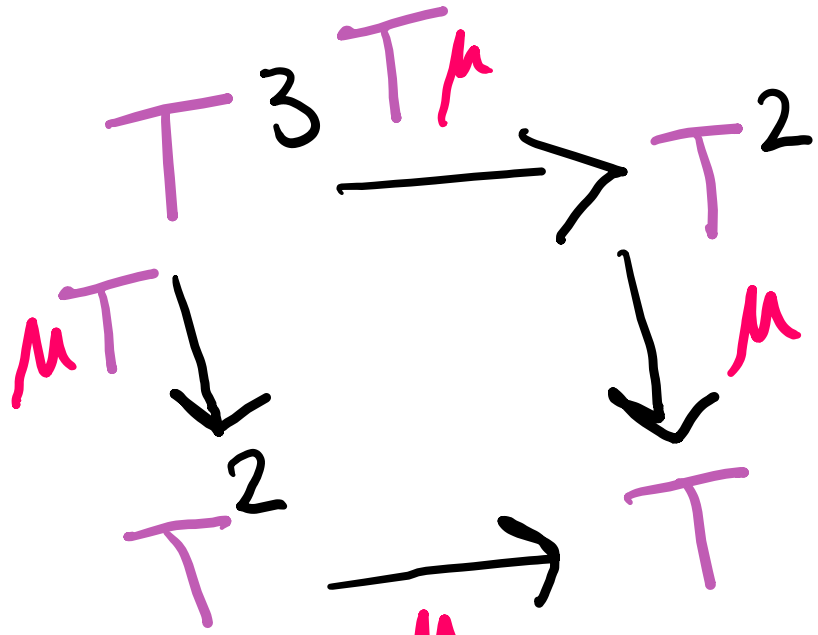
Back up, Linear exponential huh?

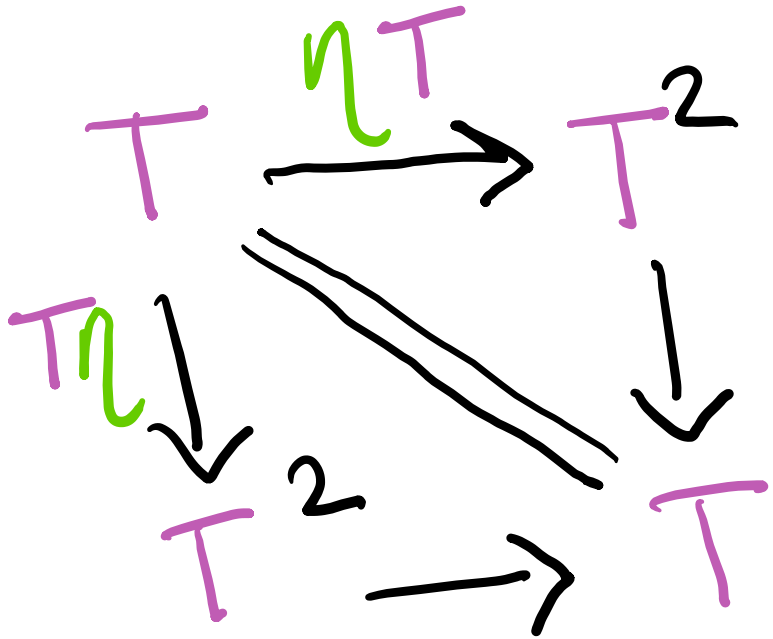
on a category $\mathcal{C}$ a monad is (T, η, μ) s.t.

$T : \mathcal{C} \to \mathcal{C}$

$\eta : id_{\mathcal{C}} \to T$

$\mu : T^2 \to T$





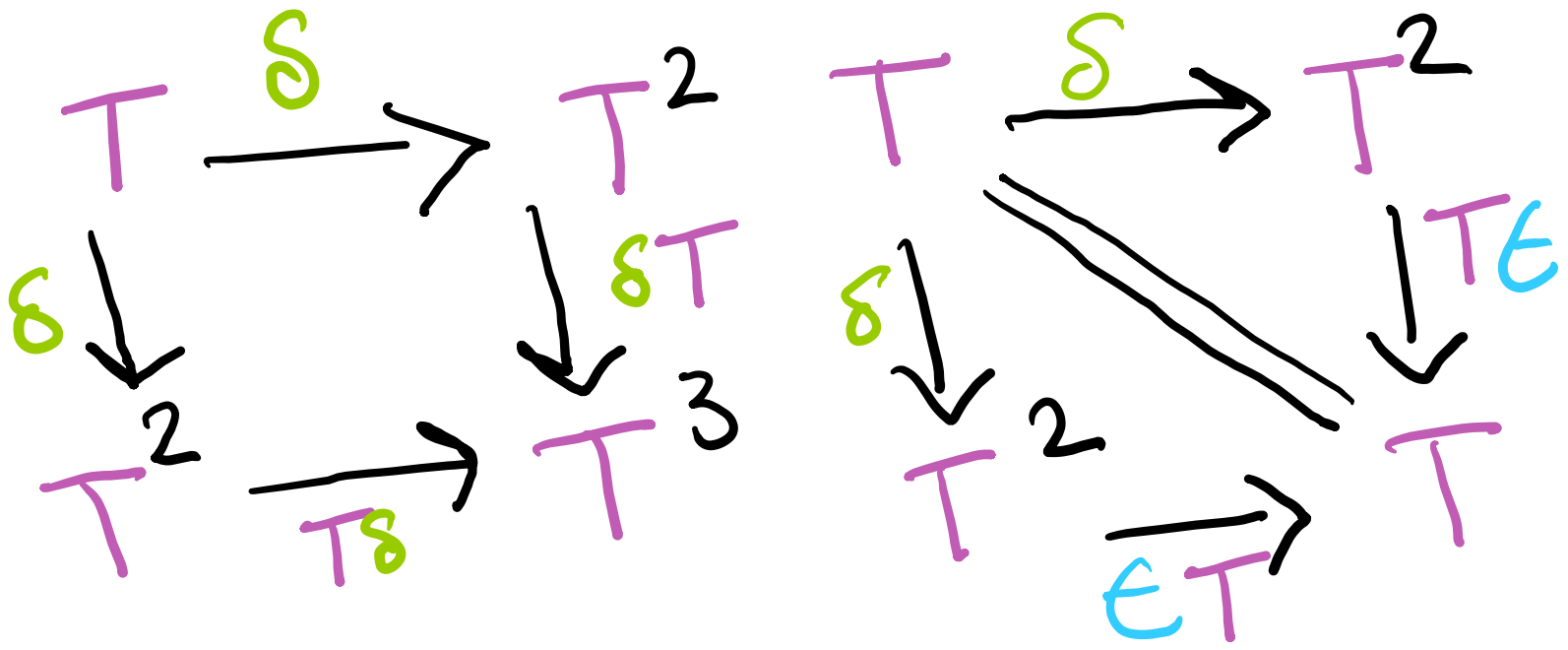
Back up, Linear exponential huh?

on a category $\mathbb{C}$ a comonad is (T, ϵ, δ) s.t.

$T : \mathbb{C} \to \mathbb{C}$

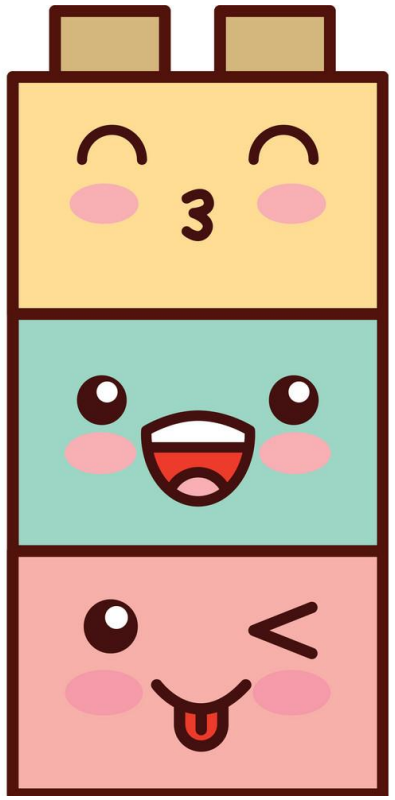
$\epsilon : T \to id_{\mathbb{C}}$

$\delta : T \to T^2$

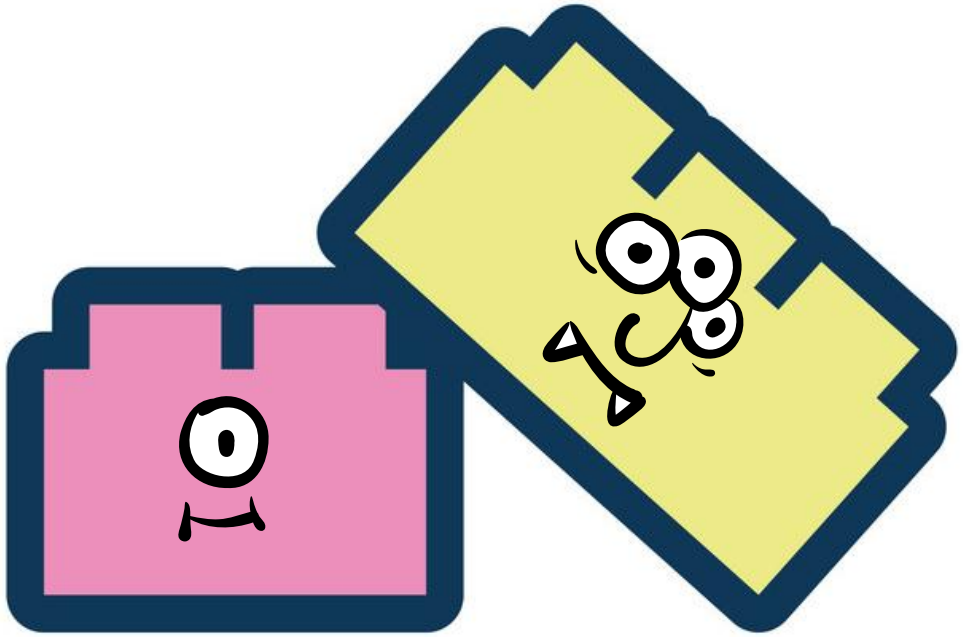


Co 21

Back up, Linear exponential huh? Continued



free algebra

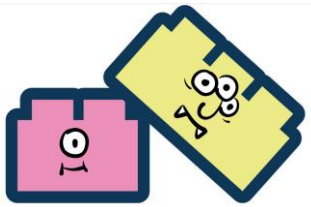


Cofree coalgebra

Back up, Linear exponential huh?

a lax Monoidal comonad where
(Preserves the Monoid structure)

every cofree !-coalgebra functor



Carries the structure of the comonand with it.

How do you even "Mix" a Logic?

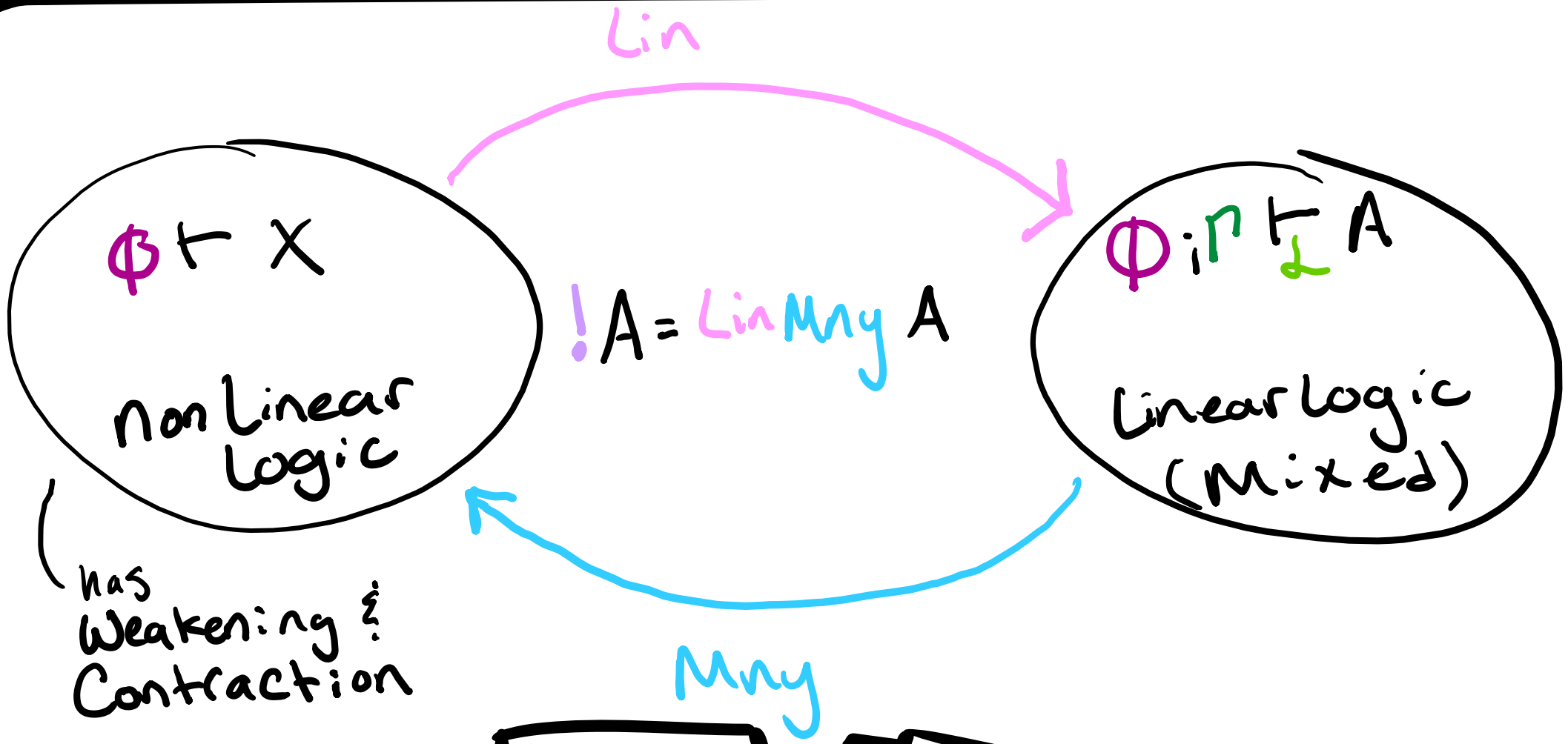
given $F \dashv G$ with counit ϵ

and Unit η we make the comonad:

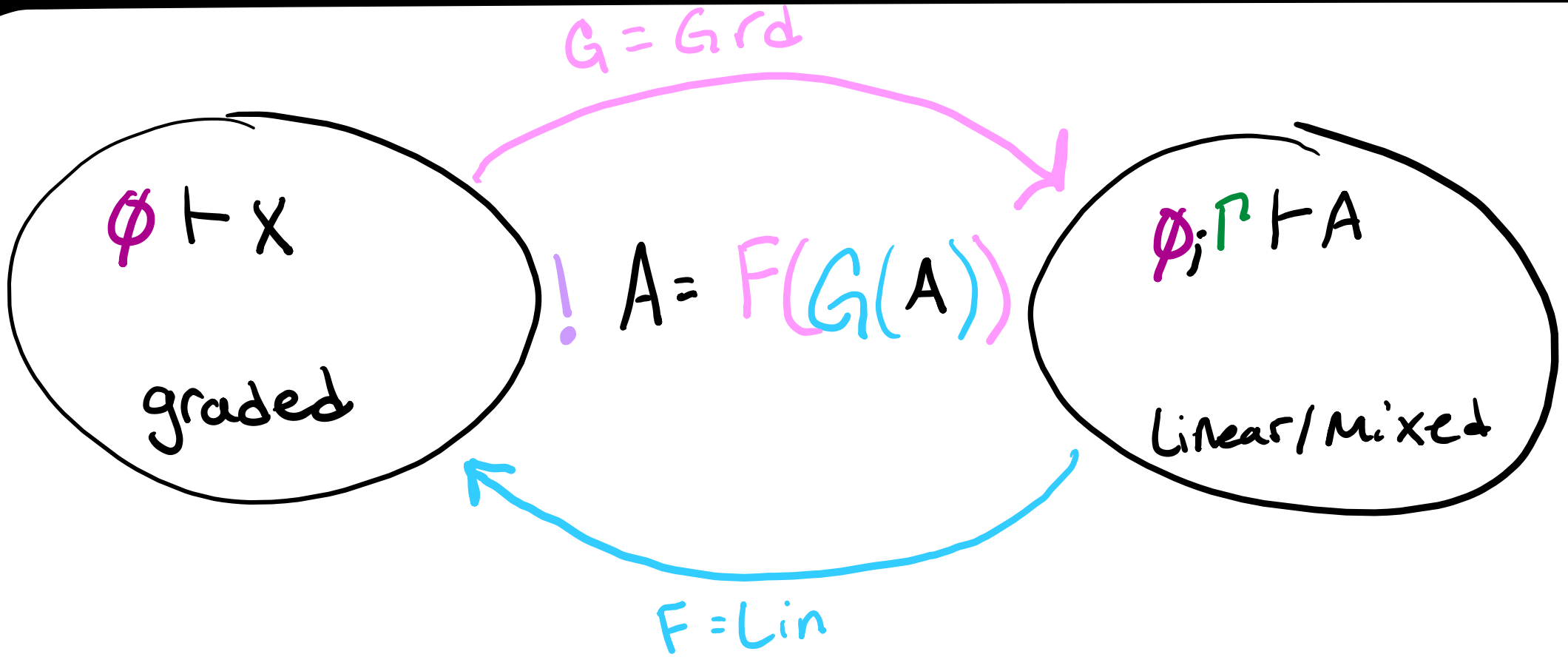
$$(FG, F\eta G, \epsilon)$$

and show it is a linear exponential Comonad

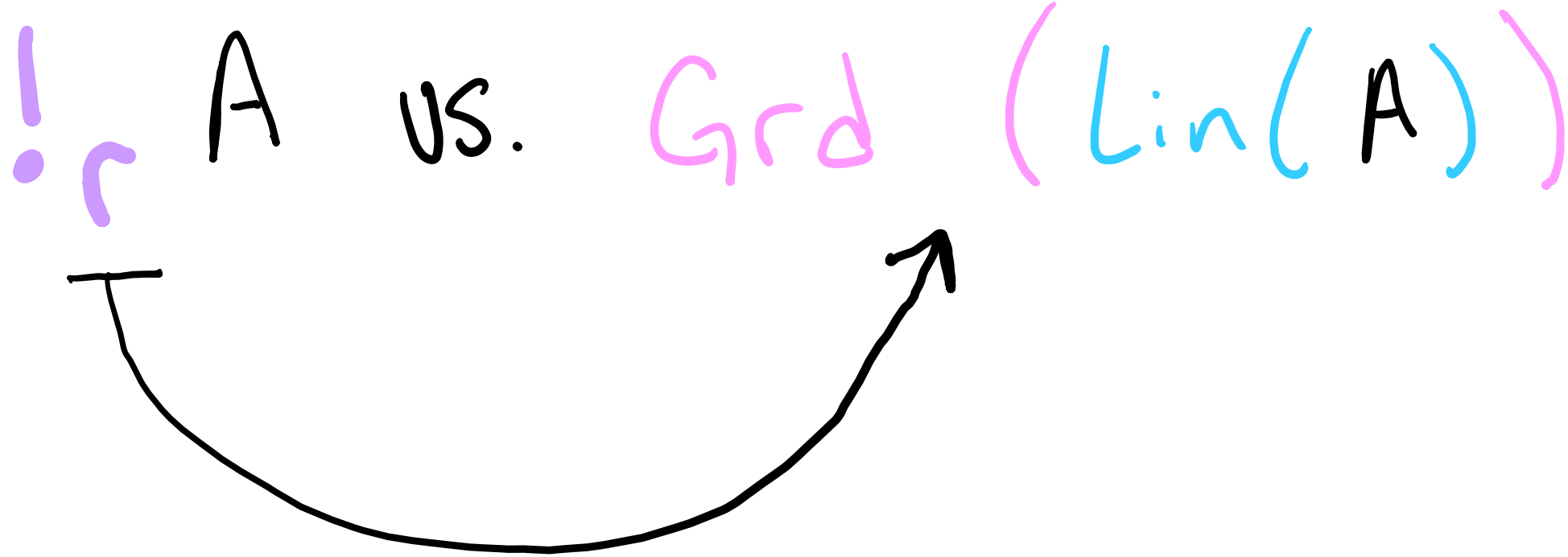
What's that got to do with logic?



What about with grades?

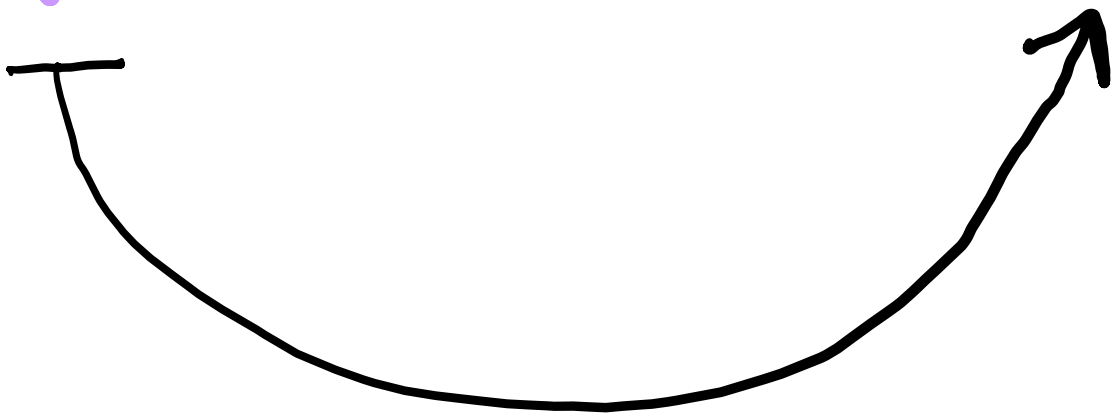


But, G(F(A)) is missing the grade

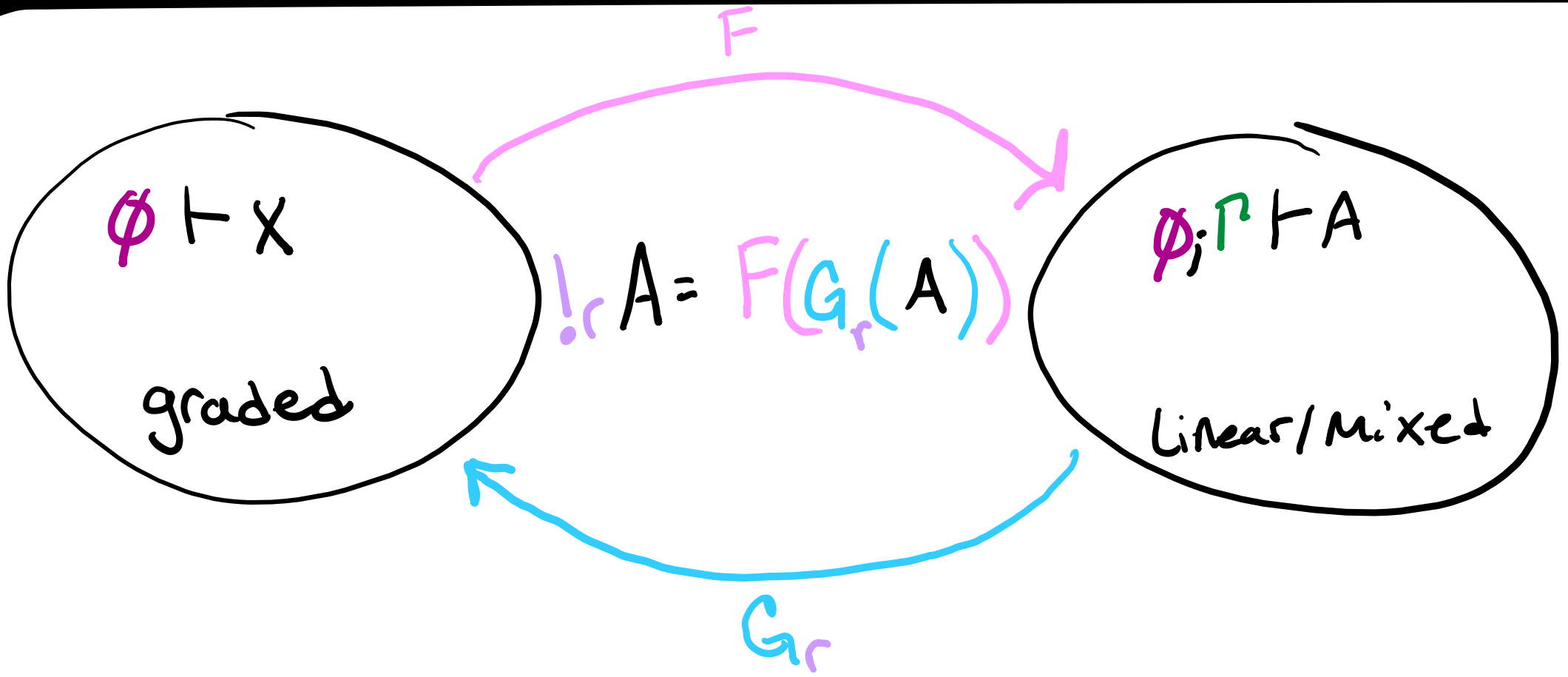


But, $G(F(A))$ is missing the grade

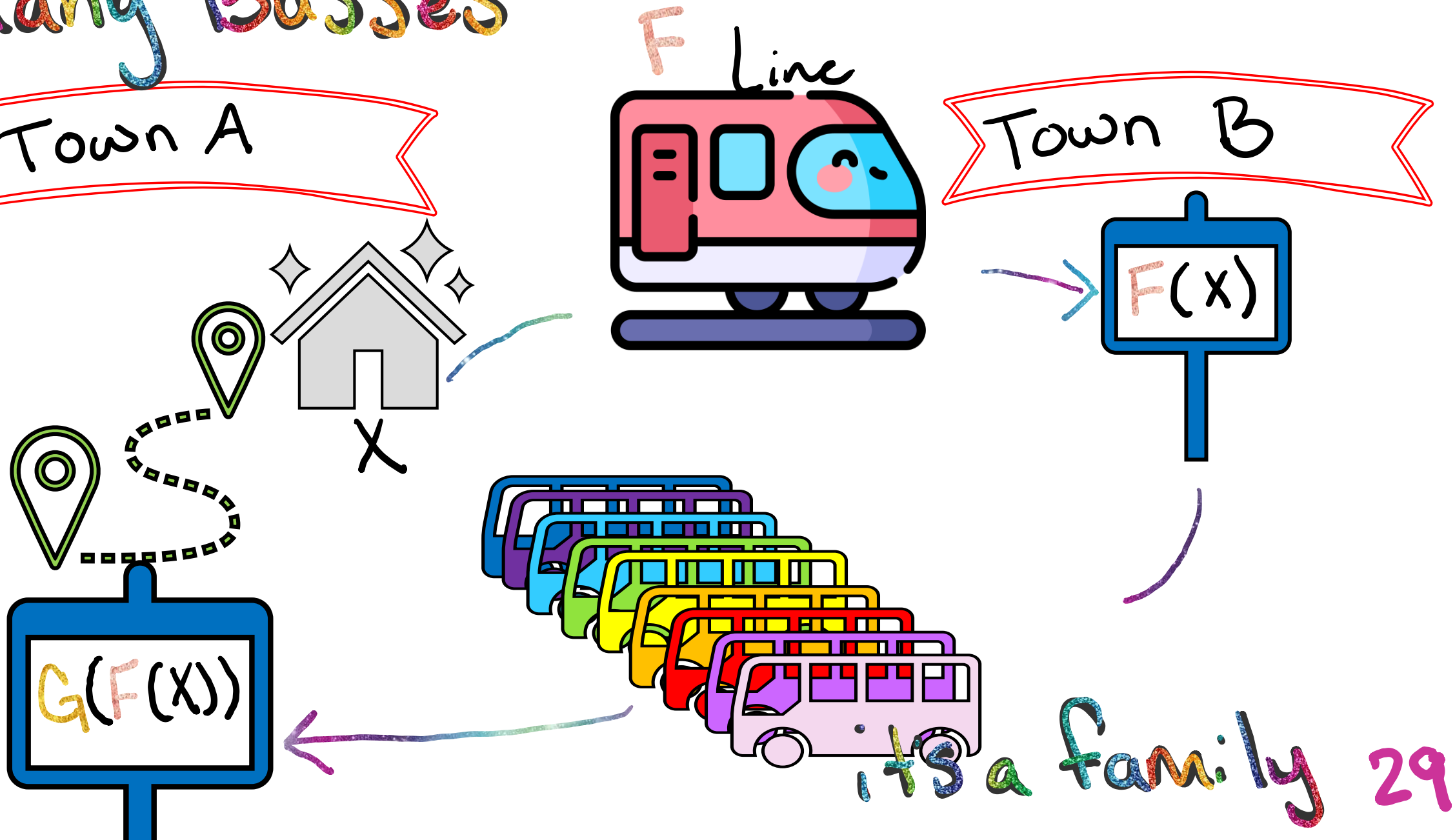
$$!_r A = \mathrm{Grd}_r(\mathrm{Lin}(A))$$



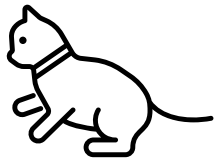
What about with grades?



Many Busses



But what does it all mean?



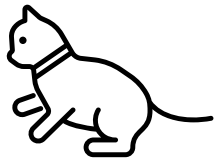
Want

Adjunction

need

Categorical model

But what does it all mean?



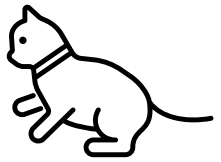
Want

Adjunction → Modality

Need

Categorical Model → easy to talk about structure

But what does it all mean?



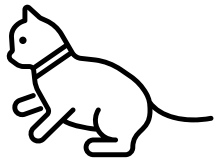
Want

Adjunction → ! Linearexponential Comonad

Need

Categorical Model → easy to talk about structure

But what does it all mean?



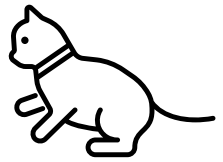
Want

Adjunction → Graded exponential comonad

Need

Categorical Model → easy to talk about structure

But what does it all mean?



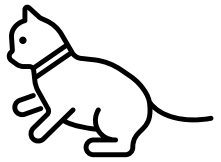
Want

Adjunction → Graded exponential comonad

need

Categorical model → Multicategory

But what does it all mean?



Want

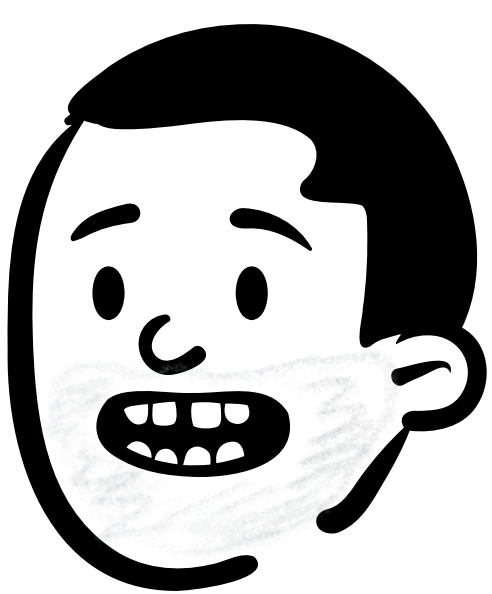
Adjunction → Graded exponential comonad

Need

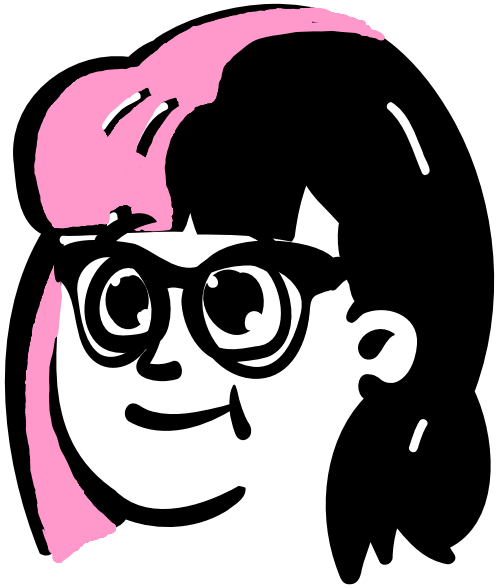
Categorical model → Graded multicategory

Thanks!

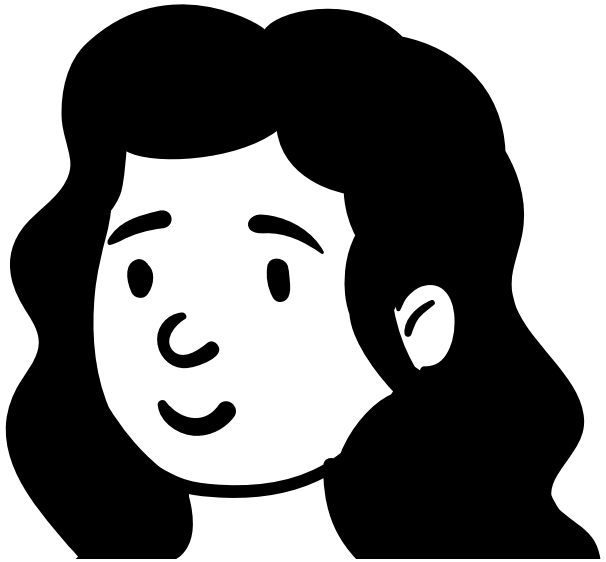
1 ⊙ Augusta University, 3 ⊙ University of Kent ⊢



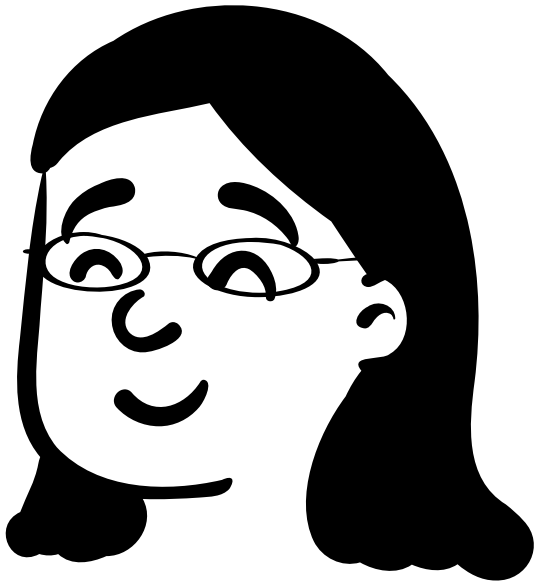
Harley Eades III



ME



Danielle Marshall

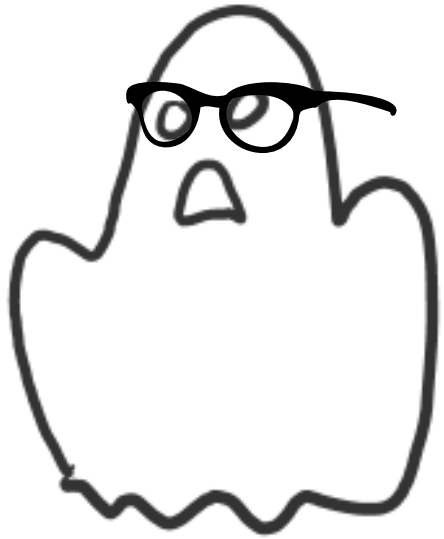


Dominic Orchard

Thanks!

1 ◦ Augusta University, 3 ◦ University of Kent ⊢

Harley Eades III



ME

Danielle Marshall

Dominic Orchard